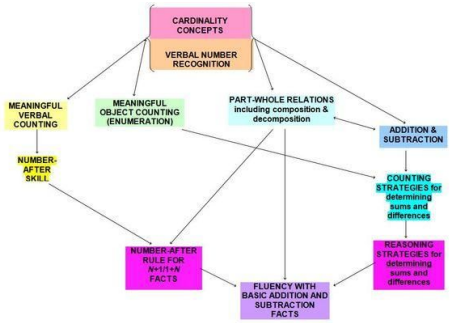
Learning Trajectory Project

Due: December 5/12, 2016

I welcome drafts at any time along the development of your learning trajectory for me to provide feedback.

Name: *Karen, Kate, and Sarah*



A trajectory toward understanding . . .

"How do you know your students are learning?" When this question was posed to me early in my teaching career, I didn't have a good answer. I fumbled around for a few moments and eventually pointed to some final products. I knew that my students didn't suddenly go from a point of not knowing to a place of understanding, but that space in between was just kind of... fuzzy to me."

--Dr. Gemma Mojica, assistant professor of mathematics education
(http://thewell.web.unc.edu/2011/10/31/trajectory-toward-understanding/)

What is a learning trajectory?

A learning trajectory is generally defined as a content-specific learning path, a developmental progression, and/or a building of conceptual components. Learning trajectories are unique to both subject and content. Dr. Mojica identifies a learning trajectory as "a description of the order of a network of experiences an individual encounters in order to move from informal ideas to more refined, complex understandings." She also points out that "a learning trajectory is not comprised by a single path. Instead, it is a network of paths that can lead to a more refined and complex understanding."

Think about the pieces of information a student would need to successfully complete a multi-digit subtraction problem requiring regrouping. That network of experiences is the learning trajectory for multi-digit subtraction. Think about a pathway that takes students from an introduction to the concept (built on essential prerequisites) to building understanding and connections, to solidifying understanding, to understanding and using the concept in specific and in generalized situations. Ultimately, a learning trajectory is a valuable tool that teachers can use to inform their instructional tasks and planning.

Overarching goals of the project: This project is intended as an opportunity for you to draw on the knowledge and abilities you are developing to develop a learning trajectory for a specific geometry concept or a specific fraction concept. You will:

Part 1: Identify Mathematical Goals and Unpack the Related Standards

This involves identifying the "big ideas" which are "clusters of concepts and skills" that are mathematically central to student learning.

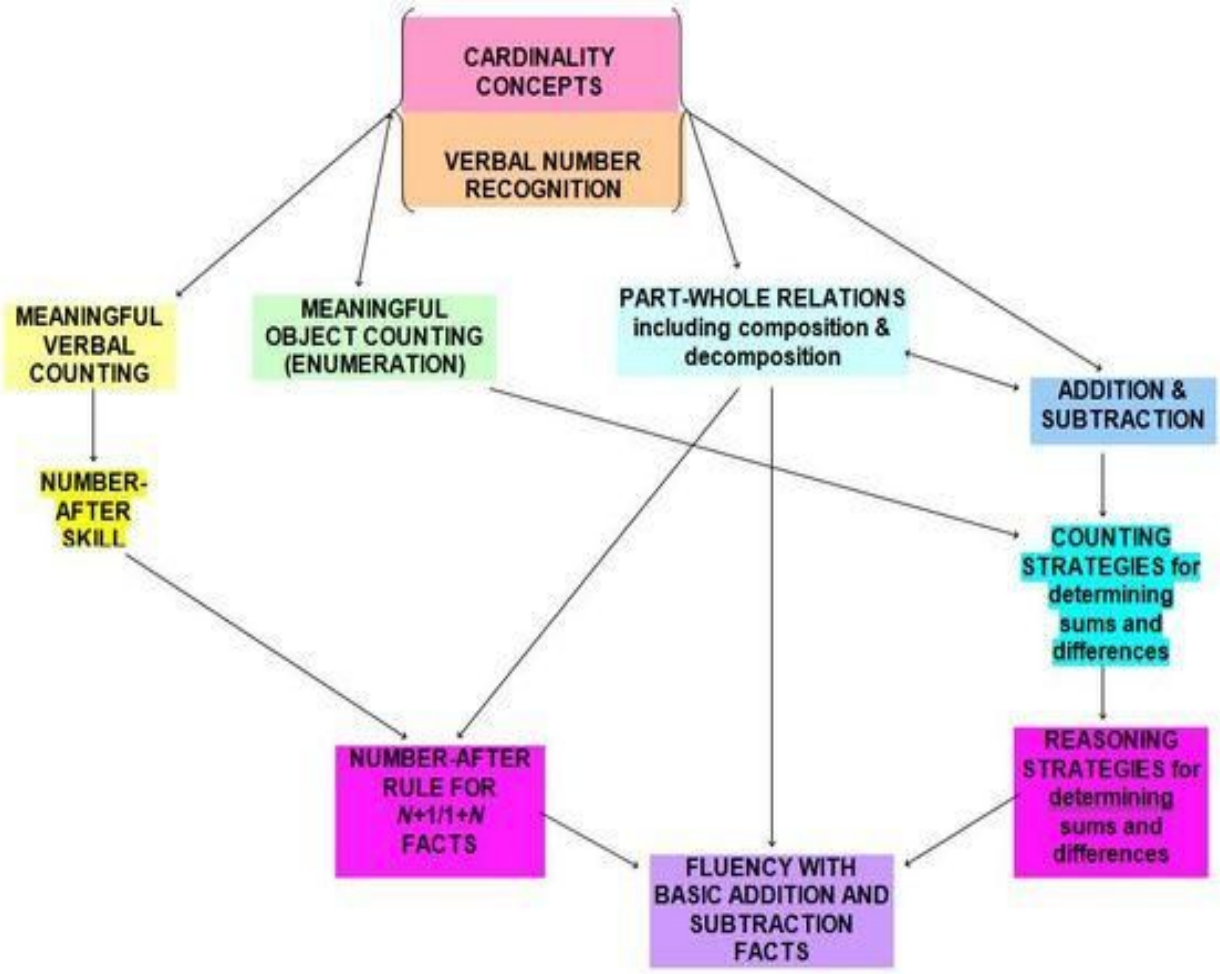
Part 2: Describe a Developmental Progression—Paths of Learning

The developmental progression/pathway describes a typical path that children follow in reaching an understanding and skill related to the mathematical goals. Each step represents more sophisticated levels of thinking, knowing, and doing than the last. Develop a meaningful pathway of experiences to go from introducing the concept to a more refined understanding of the concept.

Part 3: Design Instructional Task—Paths of Teaching

This set of instructional activities, matched to each of the levels of thinking in your path, reflects the kinds of encounters a student should have at each point in the learning trajectory.

Assessment Information: Assessment and evaluation of this project will be based on the thoroughness of your work, the extent to which you make connections between the different pieces of the learning trajectory project, the degree to which you provide rationale and theoretical support for your selections of content, goals, and learning experiences. Each part of the learning trajectory project will be assessed separately so your grade for this project will be the sum of the parts. A formal rubric will be included with each individual section of the unit.



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Topic of Study

Group Members: Kate, Karen, and Sarah

Grade Level/Topic:	Title of the Topic of Study
Second grade multi-digit subtraction	Multi-digit subtraction

Grade Level Critical Areas ● What are the critical areas for this grade level?	In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Big Idea(s)(Enduring Understandings)

- (The "big Idea" is the central concept . . . It continues through the study of the topic to the final assessment)
- Subtracting within 100.
- SUbtracting within 1,000.
- Different solving strategies for subtracting higher numbers.

Essential/"Messy" Question(s) **Provides relevant questions that foster inquiry, understanding, and transfer of learning [connects content standards and concepts and practice standards in seeking answers to question(s)]**

- What is the process for solving subtraction problems within 100? Within 1,000?

Content Strand ● What is the strand or strands that are the focus of the big ideas? ● Include the code numbers and the verbal description.	Strand: OPERATIONS AND ALGEBRAIC THINKING (2.OA) ● Represent and solve problems involving addition and subtraction (Standard 2.OA.1). Fluently add and subtract within 20 (Standard 2.OA.2) and work with equal groups of objects to gain foundations for multiplication (Standards 2.OA.3–4).
Standard Number(s) and Descriptions ● Identify the clusters of standards that illuminate the concepts in the "big ideas." ● Include the code numbers and the verbal description.	Standard 2.OA.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, *for example, by using drawings and equations with a symbol for the unknown number to represent the problem*. Standard 2.NBT.7: Add and subtract within 1,000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, and ones and ones, and that it is sometimes necessary to compose or decompose tens or hundreds.

Concepts and Skills

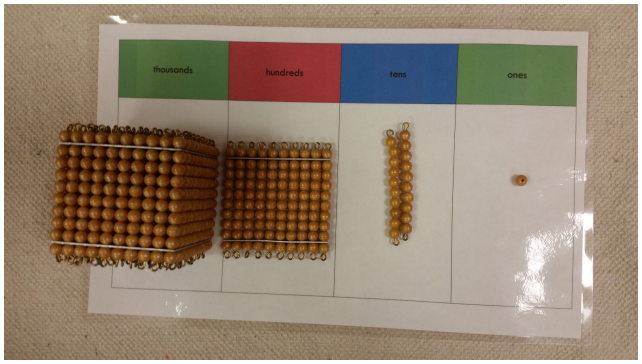
What concepts and skills must a student know/have in order to master these standards and gain enduring understanding of the "big ideas?"

Keep in mind Webb's Depth of Knowledge (DOK) levels: 1-recall; 2-skill/concepts**;** 3-strategic thinking**;** 4-extended thinking

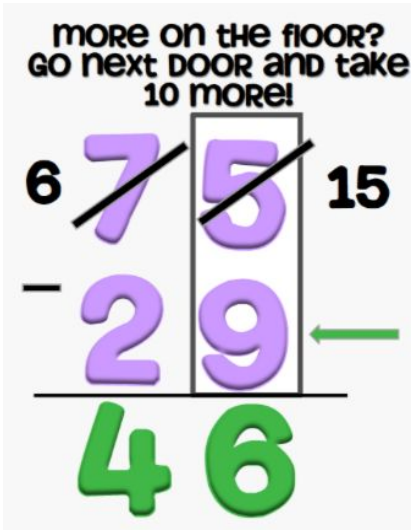
Keep in mind Multiple Modes of Representations: pictures/graphs/tables/charts; written symbols/equations, etc.; oral language; real-world situations; manipulative models
Keep in mind a Developmental Approach: concrete/experiential; pictorial/representational; symbolic/math language

Concepts students should know (list nouns)

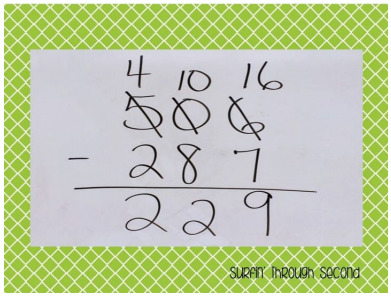
- **Place value**



- **Regrouping *****



- **Concept of zero**

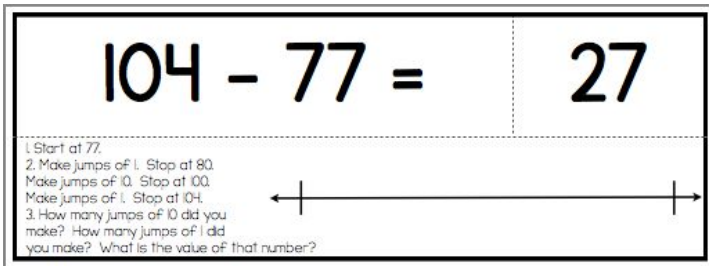


Skills students should demonstrate (list **verbs**)

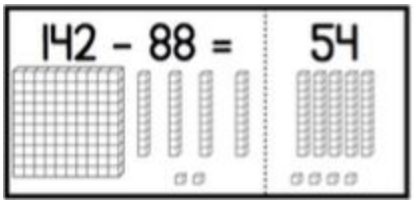
- **Explain the connection between algorithm and base ten blocks**
- **Subtraction with and without regrouping**
- **Subtraction with and without carrying over numbers**.

Contexts: models, connections, manipulatives, representations, situations, . . . students should be able to work with and within

- **Measurement Model with Number line**



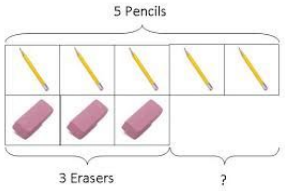
- **Base 10 blocks**



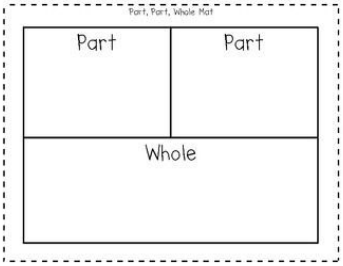
- **Word Problems**

2. Judy buys a case of lollipops. A case contains 144 lollipops. She immediately gives 32 pops to Mrs. Birkstorm's class. Judy remembers that she needs to save 42 lollipops for Principal McNamara. How many more lollipops can she give out and still have enough left over for Principal McNamara?

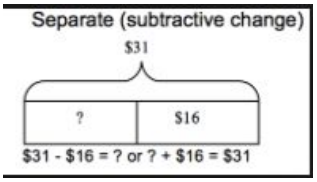
  - **Compair**



  - **Part-part-whole**

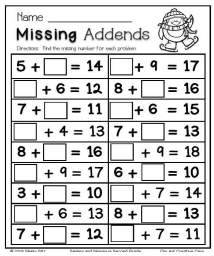


  - **Separate**

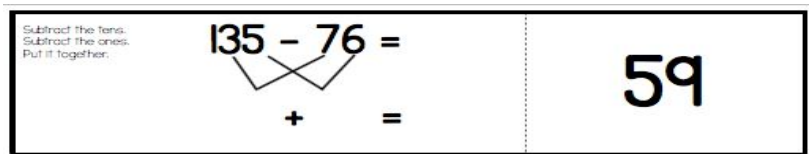


- **Working with addition (missing addend) using addition to check answers**

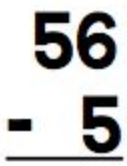
_-16=2	__-8=5
__-18=2	15-__=5
__-12=6	12-__=4
13-__=9	15-__=8
14-__=8	11-__=3
16-__=4	17-__=2



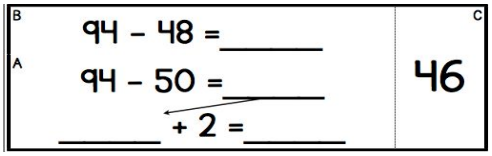
- **Break apart**



- **Vertical Subtraction**

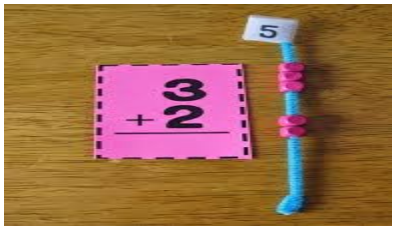


- **Rounding**



- **Real-Life**

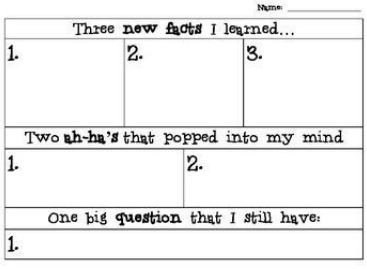
- **Symbolic**



Learning Outcomes of the Topic of Study:
Students will . . .
Be able to solve multi-digit subtraction problems in multiple contexts.

End of Unit Assessment and Formative Assessment Ideas (provide brief descriptions here and attach details as an appendix)

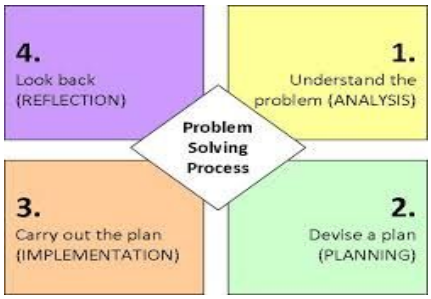
- How will students demonstrate what they know?
  - Regrouping across zero using the base ten blocks.
  - Regrouping across zero using a standard algorithm.
  - Take apart a number into place values- 5,451 = 5 thousands, 4 hundreds, 5 tens, and 1 unit.
- Through what authentic performance tasks (project . . .) might students demonstrate what they have learned or can do, as a result of this unit of study?
  - Create their own problems and challenge other students to solve them. Design a class book where students add their illustrated story problems. Make this book available to all students.
  - Have students connect this concept to the real world.
- How might you solicit student interest?
  - Have students create their own problems
  - Tie the problems to their lives and use their names
  - Present a task that will require the use of complex subtraction.
- What are some formative assessment ideas you might use?
  - Exit tickets after the lesson



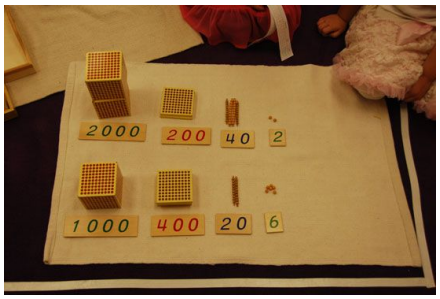
  - Asking students to talk through their thinking while solving a problem.

Instruction Includes:

- Focus on developing conceptual understanding
  - Making sure we clarify all instructions and make them developmentally appropriate.
  -
- Focus on the learning process, not just content
  - Using Polya's problem solving techniques to guide their thinking

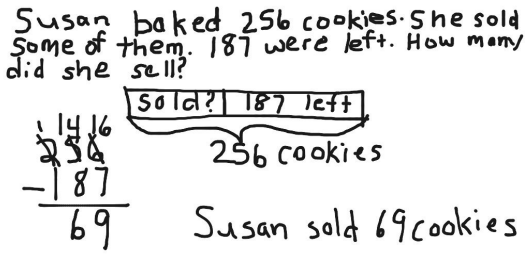


  - Have them explore using manipulatives

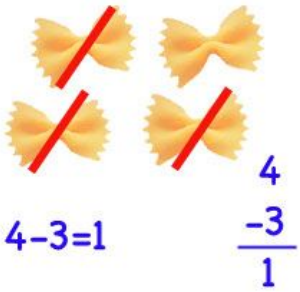


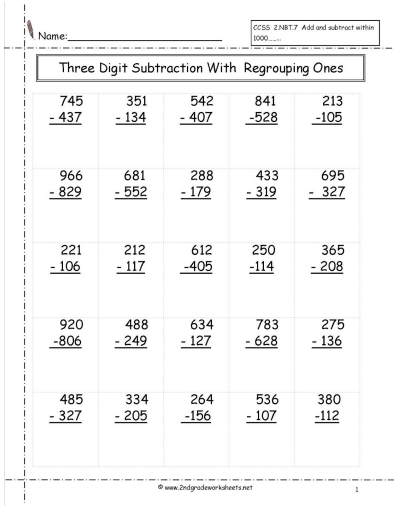
- Inclusion of appropriate high-level tasks to develop depth of understanding

- Ask students to diagram their thought process



- Observing while students are working on problems individually.
- Making sure that students can represent their thinking using a model to clearly represent thinking. Students will show understanding even when they are not around to explain (aka homework).




Name:_____ CCSS: 2.NBT.7 Add and subtract within 1000.....

Three Digit Subtraction With Regrouping Ones

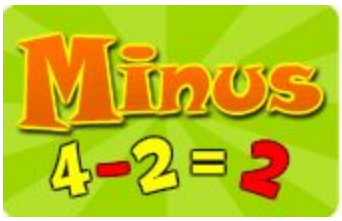
745 - 437	351 - 134	542 - 407	841 - 528	213 - 105
966 - 829	681 - 552	288 - 179	433 - 319	695 - 327
221 - 106	212 - 117	612 - 405	250 - 114	365 - 208
920 - 806	488 - 249	634 - 127	783 - 628	275 - 136
485 - 327	334 - 205	264 - 156	536 - 107	380 - 112

- Building on what students know
- Bring in funds of knowledge
- Connecting to their interests
- Having students model and teach to show their high level thinking

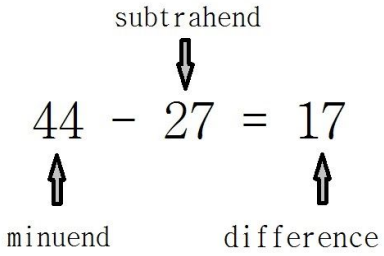
- Inclusion of appropriate low- and high-level tasks to develop proficiency and solidify understanding of concepts and skills
- Orchestrate discussions to allow "math talk," opportunities to make student thinking visible, make connections . . .
- Tasks give students time to engage in "doing" math (consider the CC Practice Standards)
- Connections to . . . .
  - Real world
  - Other subjects
  - Personal life
  - Fund of Knowledge
  - Interests
- Balance of teamwork
  - Pairing students together that work well.
  - Giving students prompts to figure out problems in groups.
  - Assigning each student to be in charge of explaining a portion of the problem.

Key Vocabulary/Symbols

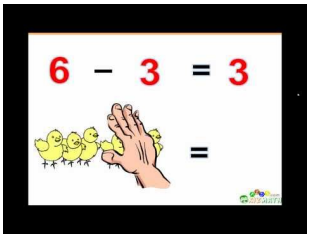
- Minus: subtracting, taking away, or decreasing by.



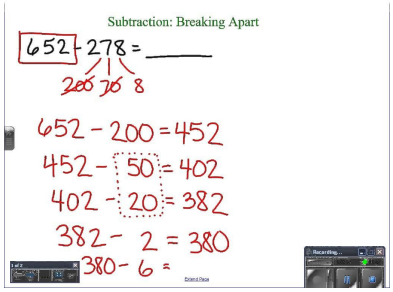
- Difference: how much one number is taken away from another.



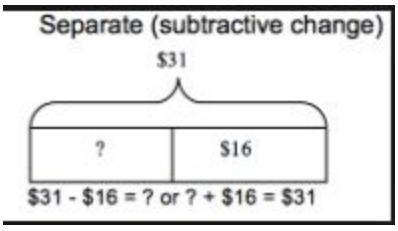
- Subtraction: Taking one number away from another.



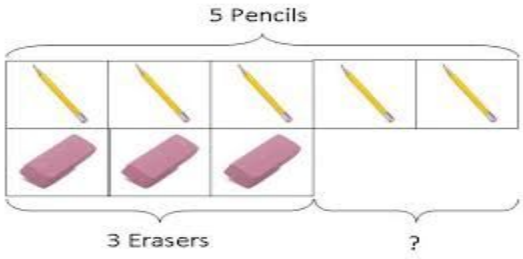
- "Take apart"



- Separate model / "Taking from"

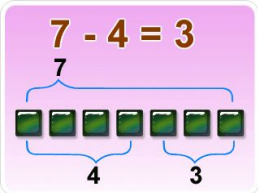


- Compare model



- Minuend

- Subtrahend
- Subtraction number sentence (equation)



Coherence: Prior/Prerequisite Grade-Level Standards	• **Standard 1.NBT.5** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
	• **Standard 1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. For example, use objects, drawings, and equations with a symbol for the unknown number to represent the problem
	• Standard 1.OA.4 Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.
	• **Standard 1.OA.6** Add and subtract within 20.
	  ○ a. Use strategies such as counting on; making ten (for example, 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (for example, 13 - 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (for example, knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (for example, adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
	  ○ b. By the end of Grade 1, demonstrate fluency for addition and subtraction within 10
	• **Standard 1.OA.8** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For

	example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + 6 = ? ● **Standard 1.NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: ○ a. 10 can be thought of as a bundle of ten ones, called a "ten." ○ b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. ○ c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). ● **Standard 1.NBT.3** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. ● **Standard 1.NBT.5** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. ● **Standard 1.NBT.6** Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. ● **Standard 2.OA.2** Fluently add and subtract within 20. ○ a. Add and subtract within 20 using mental strategies such as counting on; making ten (for example, 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (for example, 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (for example, knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (for example, adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). ○ b. By the end of Grade 2, know from memory all sums of two one-digit numbers. ● **Standard 2.NBT.1** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; for example, 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: 18 ○ a. 100 can be thought of as a bundle of ten tens called a "hundred." ○ b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). ●

Coherence: Related Grade-Level Standards	● **Standard 2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, for example, by using drawings and equations with a symbol for the unknown number to represent the problem. ● **Standard 2.OA.2** Fluently add and subtract within 20 ● **Standard 2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. ● **Standard 2.NBT.1** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; for example, 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases. ● **Standard 2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. ● **Standard 2.NBT.7** Add and subtract within 1,000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, and ones and ones, and that it is sometimes necessary to compose or decompose tens or hundreds. ● Standard 2.NBT.8 Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. ● Standard 2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects
Coherence: Subsequent Grade-Level Standards	● Standard 4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. ● Standard 4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare

	two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. ● Standard 4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Anticipated Student Difficulties and Misconceptions:

- Going backwards starting from left to right when regrouping.
- Students becoming confused with place value- renaming numbers in subtraction. Giving students opportunities to decompose numbers in many ways, then write and say the numerals helps with confusions in this area.
- Understanding regrouping as a process of decomposing the number in a place value to allow for subtraction (ie: taking 1 hundred and decomposing it into 10 tens.)
- Figuring out what piece of the question is missing (minuend, subtrahend or difference)- there are many graphic organizers including the bar system that will help students build an equation, especially when using a word problem.

Suggested Resources:

- Utah State Office of Education—Elementary Mathematics (K-5)
- Progressions Documents for CCSSM: Number and Operations in Base Ten
- Arizona Department of Education—Math Standards and "Place Mats"
- Parent Roadmaps to the Common Core Standards
- Parents' Guides to Student Success
- Progressions Documents—Main Site
- Math Solutions founded by Marilyn Burns

Further Reading

- Subtraction games for students in 2nd grade. This will help hone in on thier subtraction skills and get the opportunity to practice. http://www.education.com/activity/second-grade/subtraction/
- Subtraction methods specifically focused toward the UEN core standards. http://blogs.edweek.org/edweek/curriculum/2014/11/common-core_subtraction_many_m.html
- Parent resources for help their child at home with math in second grade. http://www.wfsd.k12.ny.us/wfdocs/files/commoncore/math-guide-second.pdf

Unpacking of Standard
Rubric

Components	9-10	7-8	<6
Completion of All Parts o Content Standard Information o Critical Areas for Grade Level o Coherence o Standards for Mathematical Practice o Key Terms Identified and Defined o Anticipated Student Difficulties and Misconceptions o Concepts/Skills/Contexts/Facts o Assessable Instructional Objectives with Examples o Use of Resources and Identification of Further Reading	All	Missing no more than 1-2	
Clear Understanding of Mathematical Concepts (illustrates depth, breadth, accuracy and includes appropriate representations and details for developmental levels)	Accurately demonstrates a detailed and thorough understanding of the concepts	Demonstrates a good understanding of the main topics; only minor inaccuracies or lack of depth, breadth	Demonstrates a partial understanding of the main concepts
Appropriate Use of Explanations and Examples (multi-modes of representation)	Uses words, diagrams, models appropriately throughout	Uses words, diagrams, models with most examples and explanations	Minimal use of words, diagrams, models and/or inappropriate use
Identification and Explanation of Vocabulary	Key terms and symbols identified and accurately defined	Most key terms and symbols accurately identified and defined	Minimal identification of terms and symbols and/or lack of accuracy
Accuracy of Analysis and Unpacking of the Standard	Thorough analysis of the standard; accurate and detailed	Analysis of the standard is evident, mostly accurate and detailed with some need for improvement	Little evidence of analysis or accuracy is minimal; needs more work
Organization and Accessibility of Information	Well organized; clear developmental trajectory	Good organization;	well
List of Resources Used	Included at least 3 additional resources	Included 1-2 additional resources	No additional resources identified
